Consider the setup of a single slit experiment. The wavelength of the incident light is \( \lambda = 440 \text{ nm} \). The slit width and the distance between the slit and the screen is specified in the figure.

Find the position \( y_1 \) of the first intensity minimum. Use a small angle approximation \( \sin \theta = \tan \theta \).

Correct answer: 5.775 mm.

Explanation:

Let: \( \lambda = 440 \text{ nm} \), \( L = 8.4 \text{ m} \), and \( a = 640 \mu \text{m} \).

\[
\delta \equiv a \sin \theta \approx a \left[ \frac{y}{L} \right]
\]

For single slit diffraction, destructive interference occurs when \( \frac{a}{2} \sin \theta = \frac{\lambda}{2} \), or simply when, \( \delta \equiv a \sin \theta = \lambda \). Thus, between the two end rays which correspond to the first minimum, the phase angle difference is \( \beta_1 = 2\pi \) and the path length difference is \( \delta_1 = \lambda \). The small angle approximation gives us \( \frac{y_1}{L} = \tan \theta_1 \approx \theta_1 \approx \sin \theta_1 = \frac{\delta_1}{a} \), or

\[
y_1 = \frac{\delta_1}{a}L = \frac{\lambda L}{a} = 5.775 \text{ mm}.
\]

Denote the intensity on the screen at \( y_2 = 3.40725 \text{ mm} \) by \( I_2 \) and the intensity on the screen at \( y = 0 \) by \( I_0 \).

Find the intensity ratio \( \frac{I_2}{I_0} \).

Correct answer: 0.268414.

Explanation:

\[
\frac{I}{I_0} = \left[ \frac{\sin(\beta/2)}{\beta/2} \right]^2.
\]
where \( y_1 \) is the position of the first intensity minimum.

From Eq. (1),

\[
\beta_2 = \frac{2 \pi a}{\lambda} \frac{y_2}{L} = \left[ \frac{2 \pi}{440 \text{ nm}} \right] (640 \mu \text{m}) (3.40725 \text{ mm}) (8.4 \text{ m}) = 3.70708 \text{ rad} = 212.4^\circ ,
\]

in agreement with the above diagram.

Following is the alternative method.

From Eq. (1) and (2), we have

\[
\mathcal{R} = \frac{\beta_2}{\beta_1} = \frac{\delta_2}{\delta_1} = \frac{y_2}{y_1} .
\]

\( \beta_2 = \mathcal{R} \beta_1 = (0.59) 2 \pi = 3.70708 \text{ rad} \) (since \( \beta_1 = 2 \pi \)), the intensity ratio at any point on the screen is

\[
\frac{I_2}{I_0} = \left[ \frac{\sin \left( \frac{\beta_2}{2} \right)}{\frac{\beta_2}{2}} \right]^2 = \left[ \frac{\sin(\mathcal{R} \pi)}{\mathcal{R} \pi} \right]^2 = \left[ \frac{\sin(0.59 \pi)}{0.59 \pi} \right]^2 = 0.268414 ,
\]

in agreement with the intensity diagram in the question.

**003** 10.0 points

**Hint:** Use a small angle approximation; e.g., \( \sin \theta \approx \tan \theta \approx \theta \) and \( \cos \theta \approx 1 \).

Consider the setup of double-slit experiment in the schematic drawing below.

**Note:** As can be seen in the figure below, one of the double-slit interference maxima is located at the first single-slit diffraction minimum.

Determine the ratio \( \frac{d}{a} \); i.e., the slit separation \( d \) compared to the slit width \( a \).

1. \( \frac{d}{a} = 4 \) correct
2. \( \frac{d}{a} = 6 \)
3. \( \frac{d}{a} = 5 \)
4. \( \frac{d}{a} = 7 \)
5. \( \frac{d}{a} = \frac{9}{2} \)
6. \( \frac{d}{a} = \frac{11}{2} \)
7. \( \frac{d}{a} = \frac{5}{2} \)
8. \( \frac{d}{a} = 2 \)
9. \( \frac{d}{a} = \frac{13}{2} \)
10. \( \frac{d}{a} = 3 \)**

**Explanation:**

At \( y \) there is a minimum for single-slit diffraction and a maxima for double-slit interference, as noted in the question.

The first minimum for single-slit diffraction occurs when

\[
\sin \theta = \frac{\lambda}{a} , \tag{1}
\]
and the maxima for double-slit interference occur when

\[ \sin \theta = (m) \frac{\lambda}{d}, \]  

(2)

The first diffraction minimum for single-slit diffraction and the fourth double-slit interference maximum \((m = 4)\) occur at the same position \(y\), as seen in the figure below.

**Figure:** The dashed curve on the left of the screen is due to single slit interference. The dashed curve on the right of the screen is due to double slit interference. The screen position is zero amplitude and the positive direction is reflected on either side of the screen.

Since the single-slit diffraction minimum masks the fourth double-slit interference maxima, one must estimate where the fourth double-slit maxima occurs using the spacing between the double-slit interference pattern shown on the right-hand side of the viewing screen, as seen in the figure above.

Using Eq. 1 and 2, we have

\[ \frac{\sin \theta}{\lambda} = \frac{1}{a} = (m) \frac{1}{d} \]

\[ \frac{d}{a} = (4) \]

\[ = 4. \]

/* If you use any of these, fix the comment symbols.

004 10.0 points

The lines in a grating are uniformly spaced at 1530 nm.

Calculate the angular separation of the second order bright fringes between light of wavelength 600 nm and 603.11 nm.

Correct answer: 6.58059 mrad.

**Explanation:**

The equation for the \(n^{th}\) bright fringe is

\[ d \sin \theta = n \lambda, \quad \text{for } n = 0, \pm 1, \pm 2, \ldots \]

where \(\theta\) is the angular displacement describing the \(n^{th}\) fringe.

Hence the angular difference between the second bright fringe \((n = 2)\) of two different wavelengths \((\lambda_2\) and \(\lambda_1)\) of light is

\[ \theta_2 - \theta_1 = \arcsin \left( \frac{2\lambda_2}{d} \right) - \arcsin \left( \frac{2\lambda_1}{d} \right) \]

\[ = \arcsin \left[ \frac{2(6.0311 \times 10^{-7} \text{ m})}{1.53 \times 10^{-6} \text{ m}} \right] \]

\[ - \arcsin \left[ \frac{2(6 \times 10^{-7} \text{ m})}{1.53 \times 10^{-6} \text{ m}} \right] \]

\[ = 0.00658059 \text{ rad} \]

\[ \Delta \theta = 6.58059 \text{ mrad}. \]

005 10.0 points

Light of wavelength 546 nm from a mercury arc falls on a diffraction grating ruled with 27700 lines/in.

What is the angular separation between the first-order images on either side of the central maximum? (Caution: do not use small angle approximation here.)

Correct answer: 73.0882°.

**Explanation:**

For double-slit,

\[ d = \frac{2.54 \text{ cm/in}}{27700 \text{ lines/in}} = 916.968 \text{ nm} \]

\[ \sin \theta_1 = \frac{\lambda}{d} \]

\[ = \frac{(546 \text{ nm})}{(916.968 \text{ nm})} \]

\[ = 0.595441, \]
so

\[ \theta_1 = 36.5441^\circ . \]

Since we are asked for the angular separation between first-order images to either side of the central maximum, it should be

\[ \theta = 2 \theta_1 = 73.0882^\circ . \]

---

**006** 10.0 points

A beam of light is diffracted by a single slit. The distance between the positions of zero intensity \((m = \pm 1)\) is 4.1 mm.

Estimate the wavelength of the laser light. Use a small angle approximation \(\sin \theta = \tan \theta \).

Correct answer: 520.481 nm.

Explanation:

Let: \( L = 2.119 \text{ m} \), \( y = 2.05 \text{ mm} \), and \( a = 0.538 \text{ mm} \).

The distance \( y_m \) from the \( m \)th minimum to the center of the first-order maximum can be calculated from the formulae

\[ \sin \theta_m = \frac{m \lambda}{a} , \]

\[ \tan \theta_m = \frac{y_m}{L} . \]

In the small angle approximation, when \( \theta \) is in radians

\[ \sin \theta = \tan \theta = \frac{y_m}{L} = \frac{m \lambda}{a} \]

\[ y_m = \frac{m \lambda L}{a} . \]

The spacing between the minima \( m = -1 \) and \( m = 1 \) is equal to

\[ b = 2 y_1 = \frac{2 \lambda L}{a} \]

\[ \lambda = a \frac{b}{2 L} \]

\[ = \frac{1}{2} (0.000538 \text{ m}) \frac{0.0041 \text{ m}}{2.119 \text{ m}} \times 10^9 \text{ nm} \]

\[ = 520.481 \text{ nm} . \]

---

**007** 10.0 points

An unpolarized light beam with intensity of \( I_0 \) passes through 2 polarizers shown in the picture.

If \( \theta = 30^\circ \), what is the beam intensity after the second polarizer?

1. \( I_2 = \frac{5}{8} I_0 \)
2. \( I_2 = \frac{9}{16} I_0 \)
3. \( I_2 = \frac{5}{16} I_0 \)
4. \( I_2 = \frac{1}{8} I_0 \)
5. \( I_2 = \frac{3}{8} I_0 \) correct
6. \( I_2 = \frac{3}{16} I_0 \)
7. \( I_2 = \frac{1}{4} I_0 \)
8. \( I_2 = \frac{7}{16} I_0 \)
9. \( I_2 = \frac{1}{16} I_0 \)
10. \( I_2 = \frac{1}{2} I_0 \)

**Explanation:**

The beam intensity after the first polarizer is

\[ I_1 = \frac{I_0}{2}. \]

We use the formula for the intensity of the transmitted (polarized) light. Thus the beam intensity after the second polarizer is

\[
I = I_1 \cos^2 \theta \\
= \frac{I_0}{2} \cos^2(30°) \\
= \frac{3I_0}{8}
\]

---

**008 (part 1 of 2) 10.0 points**

Consider 3 polarizers #1, #2, and #3 ordered sequentially. The incident light is unpolared with intensity \( I_0 \). The intensities after the light passes through the subsequent polarizers are labeled as \( I_1 \), \( I_2 \), and \( I_3 \), respectively (see the sketch).

Polarizers #1 and #3 are “crossed” such that their transmission axes are perpendicular to each other. Polarizer #2 is placed between the polarizers #1 and #3 with its transmission axis at 60° with respect to the transmission axis of the polarizer #1 (see the sketch).

---

**Explanation:**

When the light passes through the polarizer #1 it is polarized vertically. Thus the angle between its polarization and the orientation of polarizer #2 is \( \theta = 60° \). Thus the transmitted intensity is

\[
I_2 = I_1 \cos^2 \theta \\
= I_1 \cos^2(60°) \\
= \frac{1}{4} I_1.
\]

When polarized light passes through a polarizer, the transmitted intensity is \( I_2 = I_1 \cos^2 \theta \), where \( \theta \) is the angle between the polarization of the light (of \( I_1 \)) and the orientation of the polarizer #2. Thus \( I_2 = \frac{I_1}{4} \).

---

**009 (part 2 of 2) 10.0 points**

What is the final intensity \( I_3 \)?

1. \( I_3 = \frac{3}{16} I_0 \)
2. \( I_3 = \frac{5}{32} I_0 \)
3. \( I_3 = \frac{1}{2} I_0 \)
4. \( I_3 = \frac{1}{4} I_0 \)
5. \( I_3 = \frac{1}{16} I_0 \)
6. \( I_3 = 0 \)
7. \( I_3 = \frac{3}{32} I_0 \) correct
8. \( I_3 = \frac{1}{8} I_0 \)

**Explanation:**
After the polarizer #1
\[ I_1 = \left( \frac{1}{2} \right) I_0. \]

After the polarizer #2
\[ I_2 = I_1 \cos^2(60^\circ) = \left( \frac{1}{4} \right) I_1. \]

After the polarizer #3
\[ I_3 = I_2 \cos^2(90^\circ - 60^\circ) = \left( \frac{3}{4} \right) I_2 = \left( \frac{3}{4} \right) \left( \frac{1}{4} \right) I_1 = \left( \frac{3}{4} \right) \left( \frac{1}{4} \right) \left( \frac{1}{2} \right) I_0 = \left[ \frac{3}{32} I_0 \right]. \]

Let: \( \lambda = 4.55 \times 10^{-7} \text{ m} \) and \( d = 3.84 \times 10^8 \text{ m}. \)

Applying Rayleigh’s criterion,
\[ \theta_{\text{min}} = \frac{r}{D} = 1.22 \frac{\lambda}{d}, \]
where \( r \) is the radius of the spot. So, for the diameter \( x \) of the spot we obtain
\[ x = 2r = 2 \frac{1.22 \lambda D}{d} = 2 \frac{(1.22)(4.55 \times 10^{-7} \text{ m})(3.84 \times 10^8 \text{ m})}{(3.46 \text{ m})} = 123.213 \text{ m}. \]

**011 10.0 points**

On the night of April 18, 1775, a signal was to be sent from the Old North Church steeple to Paul Revere, who was 2.32 mi away: “One if by land, two if by sea.” Assume that Paul Revere’s pupils had a diameter of 2.93 mm at night, and that the lantern light had a predominant wavelength of 685 nm.

At what minimum separation did the sexton have to set the lanterns so that Revere could receive the correct message? One mile is approximately equal to 1.609 km.

Correct answer: 1.0647 m.

**Explanation:**
The angle of resolution for the Paul Revere’s pupils is
\[ \theta_{\text{min}} = 1.22 \frac{\lambda L}{D} = \frac{d}{L}. \]

Therefore
\[ d = 1.22 \frac{\lambda L}{D} = 1.22 \frac{(685 \text{ nm})(2.32 \text{ mi})}{2.93 \text{ mm}} = 1.22 \frac{(6.85 \times 10^{-7} \text{ m})(3732.88 \text{ m})}{0.00293 \text{ m}} = 1.0647 \text{ m}. \]

**010 10.0 points**

If we were to send a ruby laser beam (wavelength 455 nm) outward from the barrel of a telescope whose diameter is 3.46 m, what would be the diameter of the big red spot when the beam hit the Moon \( 3.84 \times 10^5 \text{ km} \) away? Neglect atmospheric dispersion.

Correct answer: 123.213 m.

**Explanation:**
Consider the setup of a single slit experiment. 

Hint: Use a small angle approximation; e.g., \( \sin \theta = \tan \theta \).

Determine the height \( y_3 \), where the third minimum occurs.

1. \( y_3 = \frac{\lambda}{2a} L \)
2. \( y_3 = \frac{5\lambda}{a} L \)
3. \( y_3 = \frac{3\lambda}{2a} L \)
4. \( y_3 = \frac{\lambda}{a} L \)
5. \( y_3 = \frac{5\lambda}{2a} L \)
6. \( y_3 = \frac{9\lambda}{2a} L \)
7. \( y_3 = \frac{4\lambda}{a} L \)
8. \( y_3 = \frac{2\lambda}{a} L \)
9. \( y_3 = \frac{3\lambda}{a} L \) correct
10. \( y_3 = \frac{7\lambda}{2a} L \)

Explanation:

The third minimum occurs at \( \beta = 6\pi \), which corresponds to a path difference between two end rays:

\[
\theta = \frac{b_3}{a} = \frac{y_3}{L} = \frac{3\lambda}{a} L.
\]

A converging lens with a diameter of 49.9 cm forms an image of a satellite passing overhead. The satellite has two green lights (wavelength 505 nm) spaced 1 m apart.

If the lights can just be resolved according to the Rayleigh criterion, what is the altitude of the satellite? Correct answer: 809.933 km.

Explanation:

Given: \( d = 1 \) m, \( D = 49.9 \) cm = 0.499 m, and \( \lambda = 505 \) nm = \( 5.05 \times 10^{-7} \) m.

The angular resolution is

\[
\theta = \frac{\lambda}{D} = 1.22 \frac{\lambda}{D} = 1.22 \frac{5.05 \times 10^{-7} \text{ m}}{0.499 \text{ m}} = 1.23467 \times 10^{-6} \text{ rad}.
\]

Thus the altitude is

\[
h = \frac{d}{\theta} = \frac{1 \text{ m}}{1.23467 \times 10^{-6} \text{ rad}} = \frac{1 \text{ km}}{1 \times 10^3 \text{ m}} = 809.933 \text{ km}.
\]

A binary star system in the constellation Orion has an angular separation between the two stars of \( 1.34 \times 10^{-5} \) rad.

If the wavelength is 720 nm, what is the smallest diameter a telescope can have and
just resolve the two stars?
Correct answer: 0.0655522 cm.

**Explanation:**

Let: \( \lambda = 720 \text{ nm} \).

\[
\theta = 1.22 \frac{\lambda}{D},
\]

\[
D = 1.22 \frac{\lambda}{\theta} = 1.22 \frac{720 \text{ nm}}{1.34 \times 10^{-5} \text{ rad}} \times \frac{10^2 \text{ cm}}{10^9 \text{ nm}} = 0.0655522 \text{ cm}.
\]